Section 7.3

Example The annual wages, excluding board, of U.S. farm laborers in 1926 were normally distributed with a mean of \$586 and a standard deviation of \$97. In 1926 what percentage of U.S. farm laborers had an annual wage of

- between \$500 and \$700?

  – Find the area to the left of $z = 1.18$

  – Use Table II.

z	.00	$\cdots$	.07	.08	.09	$\cdots$
.				.		
.				.		
.				.		
1.0				0.8599		
1.1	0.8643	$\cdots$	0.8790	**0.8810**	0.8830	$\cdots$
1.2				0.8997		
.				.		
.				.		
.				.		

  – Find the area to the left of $z = -0.89$ under the standard normal curve.

  – Use Table II.

z	.00	$\cdots$	.08	.09
.				.
.				.
.				.
-0.9				0.1611
-0.8	0.2119	$\cdots$	0.2177	**0.1867**
-0.7				0.2148
.				.
.				.
.				.

- Subtract the two areas

- At least \$400.

Example The length of the western rattlesnake is normally distributed with a mean of 42 inches and a standard deviation of 2 inches. Let X denote the length of one of these snakes selected at random.

- Find $P(41 < X < 45)$.

- Find $P(X < 38)$.

Example The A.C. Nielsen Company reports in Nielsen Report on Televesion that the mean weekly television viewing time for children age 2-11 years is 24.5 hours. Assuming the weekly television viewing times of such children are normally distributed with a standard deviation of 6.23 hours,

- Determine the quartiles for the viewing times.

Example Since 1900, the magnitude of earthquakes that measure 0.1 or higher on the Richter Scale in California is distributed approximately normally, with $\mu = 6.2$ and $\sigma = 0.5$. Determine the 40^{th} percentile of the maginitude of earthquakes in California.

Interpretation of z scores If x is an *observed* value of a random variable X which has a mean of μ and a standard deviation σ, then

$$z = \frac{x - \mu}{\sigma}$$

is the number of standard deviations x is away from the mean μ.

Chebychev's Rule:

- At least 75% of the observations in any data set lie within 2 standard deviations of the mean.
- At least 89% of the observations in any data set lie within 3 standard deviations of the mean.

Empirical Rule: If the data are approximately bell-shaped, then

- At least 68.26% of the observations lie within 1 standard deviation of the mean.
- At least 95.44% of the observations lie within 2 standard deviations of the mean.
- At least 99.74% of the observations lie within 3 standard deviations of the mean.

Example A sample of 1000 students take the SAT test. The scores on the test are approximately bell-shaped with a mean of 500 and a standard deviation of 100 points.

- Approximately how many students in the sample score between 400 and 600?

- At least 950 students scores' fall in what range?

Section 7.4: Normal Probability Plots

We need a test to determine whether or not a data set comes from a population that is normally distributed.

Example The number of miles driven last year, in thousands, on a random sample of 15 cars yielded the following data:

13.2	13.3	11.9	15.7	11.3
12.2	16.7	10.7	6.3	13.6
14.8	9.6	11.6	8.7	15.0

Does this sample suggest that the number of miles driven last year is normally distributed on the population of all cars driven last year?

Expected Normal Scores The z score we would expect for each observation in the sample if the sample had come from a normal distribution.

- Arrange the data in ascending order.

- Compute

$$f_i = \frac{i - 0.375}{n + 0.25},$$

  where i is the i^{th} number in the list, and n is the sample size. f_i is the area we would expect to be to the left of the i^{th} observation in the data, had the data come from a normal distribution.

- Find the z score corresponding to f_i. That is, find the z score having an area of f_i to its left under the standard normal curve. This is the z score we would expect to correspond to the i^{th} observation, *had the data come from a normal distribution.*

Example Find the expected z scores for the mileage data.

i	**Mileage**	f_i	z score
1	6.3		
2	8.7		
3	9.6		
4	10.7		
5	11.3		
6	11.6		
7	11.9		
8	12.2		
9	13.2		
10	13.3		
11	13.6		
12	14.8		
13	15.0		
14	15.7		
15	16.7		

Normal Probability Plot A scatter plot of z-scores vs the observed values of the data set, with z-scores on the vertical axis and observed data values on the horizontal axis.

Example Make a normal probability plot of the mileage data.

Assessing Normality If the sample data are taken from a population that is normally distributed, then the normal probability plot should be **approximately linear**.

Example Does the miles driven last year appear to be normally distributed?

Using SPSS to Draw a Normal Probability Plot Exercise 7

Section 7.6 Normal Approximations to Binomial Distributions

Example A student takes a true false exam with 10 questions. The student has no clue and totally guesses on each one. Let X be the number of correct answers guessed.

- Compute the probability that the student guesses either 7 or 8 correct answers.

- Determine the probability distributuion of X.

x	$P(X = x)$
0	0.0010
1	0.0098
2	0.0439
3	0.1172
4	0.2051
5	0.2461
6	0.2051
7	0.1172
8	0.0439
9	0.0098
10	0.0010

- Graph the probability histogram of the random variable X

- What is the mean and standard deviation of X?

- Determine the area under the normal curve between 6.5 and 8.5. This is the **Correction for Continuity**.

- Determine the z scores for 6.5 and 8.5.

- Find the corresponding area under the standard normal curve.

- Compare this with the probability of guessing 7 or 8 correct answers.

Example The infant mortality rate in India is 139 per 1000 live births. Determine the probability that out of 1000 randomly selected live births in India there are between 120 and 150 infant deaths.

- Determine n and p.

- Find μ and σ.

- Make the Correction for Continuity

- Find the z scores and the appropriate area from table II.

Example In the last example, find the probability of obtaining exactly 150 infant deaths.

Example In the previous example, find the probability of obtaining at most 150 infant deaths.