Solutions to Final; Phys 185 (Edis)

1. (20 points) Explain what temperature is. I never provided you with a short definition, and there's no point to being able to memorize and repeat a definition anyway. Start with the fact that it does not make sense to talk about temperature for a collection of three particles, but that it does make sense for 10^{20} particles. Then develop this point and present what your notion of temperature now is, after having taken a college physics course.

Answer: Answers will vary. But they should start with the fact that for a large collection of particles, it becomes impossible to keep track of variables describing individual particles. We therefore need variables that capture aggregate, statistical properties of the collection. Temperature is one of these variables; you can make sense of it at this point by connecting it to energy in contexts like an ideal gas, the notion of thermal equilibrium, etc. etc.

2. (30 points) You enter an amusement park ride: a cylindrical room that spins around a central axis. Once it is spinning fast enough, the floor moves downward and you find yourself pinned against the wall without falling to the floor. Say the radius of the room is 3.0 m, the coefficient of static friction between you and the wall is 0.41, and the coefficient of kinetic friction is 0.33. Find the minimum frequency at which the room has to rotate before it is safe to remove the floor from under you.

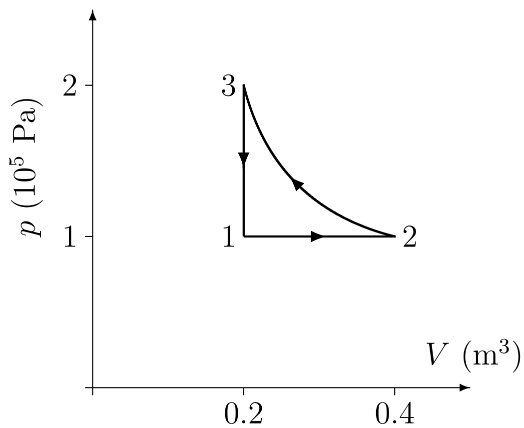
Answer: There are three forces on you: weight (downward), static friction (upward), and the normal force (toward the axis of the cylinder. Summing the forces and setting that equal to mass times acceleration,

$$\sum F_y = f_s - mg = 0 \qquad \sum F_x = n = m\frac{v^2}{r}$$

For the minimum frequency, you must be at the maximum friction force, so $f_s = f_{s,max} = \mu_s n$. We can now solve these equations for $v = \sqrt{gr/\mu_s}$, and use $f = v/2\pi r$:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\mu_s r}} = 0.45 \text{ revolutions/s}$$

3. (60 points) You have a monatomic ideal gas that goes through the cycle $1 \to 2 \to 3 \to 1$ shown in the diagram. No gas molecules are added or removed during the cycle.



(a) The $2 \to 3$ part of the cycle takes place at *constant temperature*, so $T_2 = T_3$. The area under this "isotherm" on the p-V diagram is given by:

- **(a)** $nRT_2 \ln\left(\frac{V_3}{V_2}\right)$**. It has the right units, it is 0 when $V_3 = V_2$, it becomes negative when $V_3 < V_2$ as in the diagram.**
- (b) $\frac{3}{2}nRT_2\, e^{-V_3/V_2}$
- (c) $\frac{3}{2}nRT_2 \frac{V_3}{V_2}$
- (d) $nRT_2(p_3 - p_2)$
- (e) $nRT_2(V_3 - V_2)$

Give the reason for your choice. (*Hints:* What should the result be when $V_3 = V_2$? What sign should the result be?)

(b) The temperature at point 1 is $T_1 = 160$ K. Find the work done by the gas for each step of this cycle: $W_{1\to2}$, $W_{2\to3}$, $W_{3\to1}$.

Answer: First, using $pV = nRT$, we get

$$T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} = 320 \text{ K}$$

Now the works, which are the areas under the curves:

$$W_{1\to2} = p_1(V_2 - V_1) = 20 \text{ kJ}$$

$$W_{2\to 3} = nRT_2 \ln\left(\frac{V_3}{V_2}\right) = p_2V_2 \ln\left(\frac{V_3}{V_2}\right) = -28 \text{ kJ}$$
$$W_{3\to 1} = 0$$

(c) Find the change in the gas's thermal energy for each step of this cycle: $\Delta E_{1\to 2}$, $\Delta E_{2\to 3}$, $\Delta E_{3\to 1}$.

Answer: For a monatomic ideal gas, $E = \frac{3}{2}nRT = \frac{3}{2}pV$. Therefore

$$\Delta E_{1\to 2} = E_2 - E_1 = \frac{3}{2}p_2V_2 - \frac{3}{2}p_1V_1 = 30 \text{ kJ}$$
$$\Delta E_{2\to 3} = E_3 - E_2 = \frac{3}{2}p_3V_3 - \frac{3}{2}p_2V_2 = 0$$
$$\Delta E_{3\to 1} = E_1 - E_3 = \frac{3}{2}p_1V_1 - \frac{3}{2}p_3V_3 = -30 \text{ kJ}$$

(d) Find the heat added to the gas for each step of this cycle: $Q_{1\to 2}$, $Q_{2\to 3}$, $Q_{3\to 1}$.

Answer: Use $\Delta E = Q - W$ and add up previous results:

$$Q_{1\to 2} = 50 \text{ kJ} \qquad Q_{2\to 3} = -28 \text{ kJ} \qquad Q_{3\to 1} = -30 \text{ kJ}$$

(e) Find the total heat input to this gas in one cycle, Q_{in}. Also find the total heat removed from the gas, Q_{out}, and the total work done, W.

Answer: The total heat taken in is the sum of the heats with positive signs: $Q_{in} = 50$ kJ. Heat removed is indicated by a negative sign, so $Q_{out} = 28 + 30 = 58$ kJ.

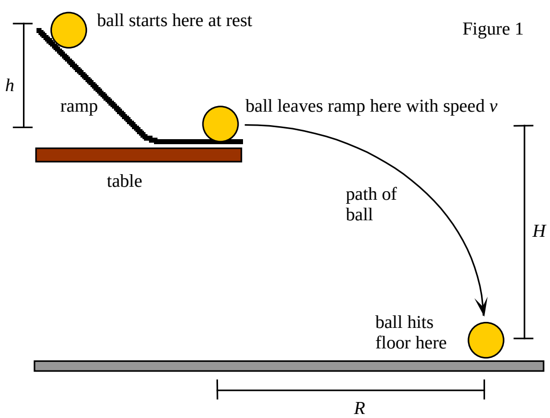
The total work is $W = 20-28+0 = -8$ kJ, the (negative) area enclosed by the cycle.

(f) Is this a heat engine or a refrigerator? If a heat engine, what is its efficiency? If a refrigerator, what is its COP?

Answer: This is a refrigerator, needing $W = 8$ kJ per cycle to remove $Q_C = 50$ kJ from what it's cooling, dumping $Q_H = 58$ kJ into a hot reservoir. Its COP is

$$\text{COP} = \frac{Q_C}{W} = 6.25$$

4. (30 points) Say you're performing your ball-and-ramp experiment (Lab 10) with two balls that look identical in every respect, with the same radius r and mass m. One ball has its mass evenly distributed, with moment of inertia $I_1 = \frac{2}{5}mr^2$. The other ball is hollow inside, with $I_2 = mr^2$. Making the usual simplifying assumptions—rolling without slipping, no air resistance—find equations for R_1 and R_2, the distance from the table for each ball. The only variables appearing in your equations for R_1 and R_2 should be h and H.



Answer: Use energy conservation for each ball, including the rotational kinetic energy $\frac{1}{2}I\omega^2$, to find the launch speed. Remember that for rolling without slipping, $\omega = v/r$.

$$mgh = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_1}{r}\right)^2 = \frac{7}{10}mv_1^2$$

$$mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}\left(mr^2\right)\left(\frac{v_2}{r}\right)^2 = mv_2^2$$

This gives

$$v_1 = \sqrt{\frac{10}{7} gh} \qquad v_2 = \sqrt{gh}$$

Now for the projectile motion part. The time to fall is Δt, with $H = \frac{1}{2} g (\Delta t)^2$, so $\Delta t = \sqrt{2H/g}$. The time traveled during this time is

$$R_1 = v_1 \Delta t = \sqrt{\frac{20}{7} hH} \qquad R_2 = v_2 \Delta t = \sqrt{2hH}$$

$R_1 > R_2$, as it should be.

5. (30 points) Take a small bubble of air at a depth d below the ocean surface. There are n moles of air in the bubble, and air is approximated very well as an ideal gas. Let's assume that the bubble is small enough that we can assume a single depth and a single pressure value accurately characterizes the bubble. Let's also assume that the ocean has a constant temperature T at any depth, and that the air is always in thermal equilibrium with the ocean. Use p_{atm} to represent atmospheric pressure and ρ_w to represent the density of water.

(a) Write down an equation for the volume of the bubble, showing how it changes with depth d.

Answer: Use $pV = nRT$, and the fact that the pressure within the bubble will be equal to the water pressure at its depth, $p = p_{atm} + \rho_w g d$:

$$V = \frac{nRT}{p_{atm} + \rho_w g d}$$

(b) Now write down an equation for the buoyancy force experienced by the bubble.

Answer: The buoyancy force is equal to the weight of water of an equal volume:

$$F_B = \rho_w V g = \frac{nRT \rho_w g}{p_{atm} + \rho_w g d}$$

(c) Make a rough sketch of the buoyancy force versus depth. Make sure the sketch is clear about whether F_B almost at the surface ($d = 0$) is zero, infinite, or a finite value.

Answer: The curve for F_B should begin with a finite value for $d = 0$, and monotonically decrease without ever becoming zero.

6. (30 points) Say we have an animal that we model as an object with a surface area of 0.34 m^2, wrapped up in a layer of fat with $k_{fat} = 0.21$ W/m·K and thickness 0.0010 m, and a layer of fur with $k_{fur} = 0.020$ W/m·K and thickness of 0.0050 m. The interior temperature of the animal is 35°C and the external temperature is −1°C. Calculate the rate of heat loss from the animal to the environment.

Answer: The rate of heat transfer through the fat and fur layers is the same. Call the temperature at the the fat-fur interface T_f. Then

$$\frac{Q}{\Delta t} = k_{fat}\frac{A}{L_{fat}}(T_{in} - T_f) = k_{fur}\frac{A}{L_{fur}}(T_f - T_{out})$$

Putting in some numbers,

$$\frac{Q}{\Delta t} = 71.4(T_{in} - T_f) = 1.36(T_f - T_{out})$$

Solving, $T_f = 34.3$°C. Putting this in, we find $Q/\Delta t = 48$ W.