College Physics I
Lab 10: Ball and Ramp

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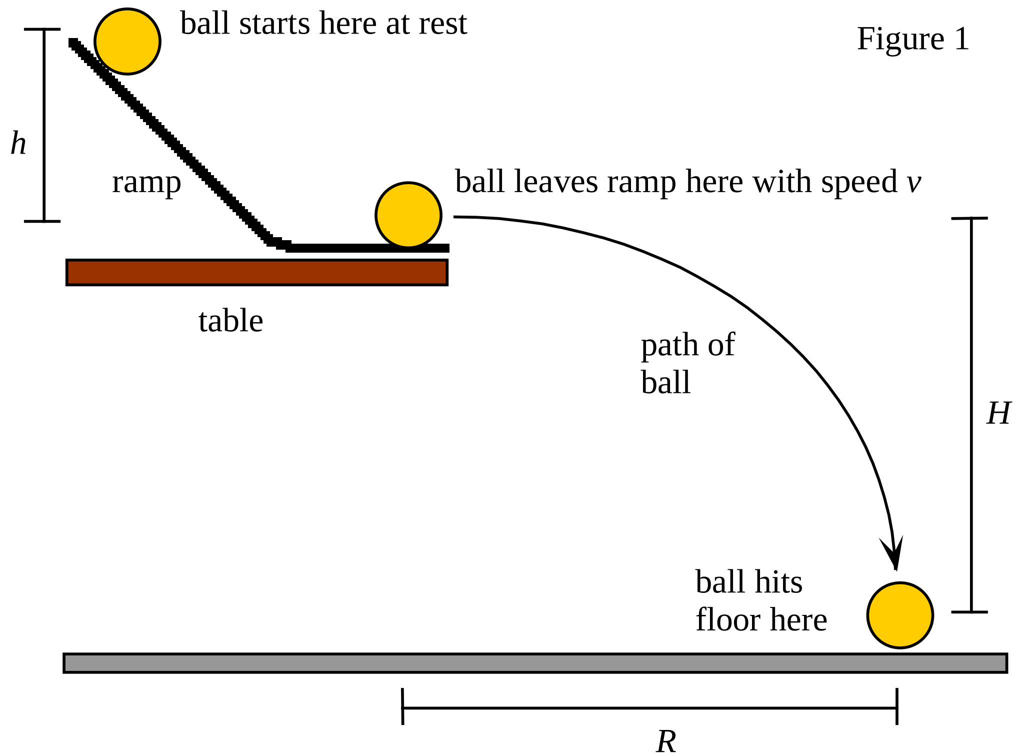

You are going to roll a ball of mass m down a ramp. The ramp will be on the edge of a table, so when the ball leaves the ramp, it will go sailing though the air and land on the floor (see Figure 1). In what follows, we will see that, from our knowledge of physics, we can predict the distance R the ball will travel if we know h and H. We will combine two concepts to get this result; one is *conservation of energy,* and the other is *projectile motion.*

The difference in the gravitational potential energy (U_g) of the ball when it is at the top of the ramp and when it is at the bottom of the ramp is $\Delta U_{\text{top to bottom}} = mgh$, where g is the acceleration due to gravity. Assume $g = 9.80\ \text{m/s}^2$. Since energy is conserved, this means that, if the ball was at rest at the top of the ramp, it will have kinetic energy (K) at the bottom of the ramp of:

$$K_{\text{bottom}} = \Delta U_{\text{top to bottom}} = mgh \tag{1}$$

To put it simply, the potential energy the ball loses as it falls from the top of the ramp to the bottom of the ramp is converted, as it falls, to kinetic energy. At the bottom of the ramp, it has all been converted to kinetic energy.

What is K_{bottom} in terms of the speed v and rotational velocity ω of the ball?

$$\begin{aligned} K_{\text{bottom}} &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2 \end{aligned} \tag{2}$$

Here, I is the moment of inertia of a sphere and r is the radius of the sphere. Recall that, just as m is resistance to change in linear motion and v is linear velocity, so I is resistance to change in rotational motion and ω is rotational veloctiy. I for a sphere of mass m and radius r is $\frac{2}{5}mr^2$. For a sphere of radius r which is rolling with rotational velocity ω without slipping, and is thus also moving linearly with speed v, $v = r\omega$. Combining Equations (1) and (2):

$$v = \sqrt{\frac{10gh}{7}} \tag{3}$$

Therefore, to the extent that energy is conserved as the ball rolls down the ramp, we can figure out how fast the ball is traveling as it leaves the ramp just by knowing the height of the ramp—regardless of the shape of the ramp!

The speed of the ball as it leaves the ramp will determine the horizontal distance R that it will travel (the range). Because of the shape of our ramps, the ball leaves the ramp with only a horizontal component of motion, that is $v_{ix} = v$ and $v_{iy} = 0$, where v is the initial speed. However, after leaving the ramp, the ball will accelerate downward, so it will have an increasing downward velocity. Since the horizontal and vertical components of the ball's motion can be treated separately, and since there is no horizontal acceleration but there is a vertical acceleration of g downward, then:

$$R = v\Delta t \tag{4}$$
$$H = \frac{1}{2}g\Delta t^2 \tag{5}$$

Combining Equations (4) and (5):

$$R = v\sqrt{\frac{2H}{g}} \tag{6}$$

Finally, combining Equations (3) and (6):

$$R = \sqrt{\frac{20hH}{7}} \tag{7}$$

The effect of error and uncertainty

This would be a pretty boring lab if all you did was roll the ball down the ramp and see if where it landed agreed with Equation (7). Instead, you are going to predict where the ball will land by drawing an ellipse on the floor, and seeing if the ball lands within the ellipse. You will get extra points for every trial that lands in the ellipse, but the bigger your ellipse is, the fewer points you will get. So you have to decide where the center of your ellipse should be, and what the size of your ellipse will be.

Equation (7) will be your starting point. But Equation (7) was derived assuming many things. For example:

- Equation (1) assumes no energy was lost to friction as the ball rolled down the ramp.
- Equation (2) assumes the ball touched the ramp at its outer most radius.
- Equation (2) also assumes the ball rolled without slipping.

- Equations (4) and (5) assume no energy was lost to friction with the air.

You need to decide how good each of these assumptions are, and if they are not so good, you need to decide how they will influence where the ball will land.

How are you going to decide how big the ellipse should be, and how different from a circle it should be? It will depend, at least partly, on how precisely you measure h and H. As a very rough rule of thumb, if h and H are measured correctly to within 5%, then you should not expect your prediction of R from Equation (7) to be significantly better than to within 5%! You will also have to think about what "imperfections" in the experiment will affect sideways motion of the ball differently than the forward motion of the ball.

Activity 1: Make a prediction and test it

Measure h and H as carefully as you are able. Choose a ball. Then, draw an ellipse on a piece of paper taped to the floor. The center of the ellipse should be where you predict the ball will land, and the size and shape of the ellipse should account for uncertainty in your prediction. Cover the paper with carbon paper (carbon side down), and then lay a piece of plain paper over that. Don't tape the carbon paper or the top piece of paper. Now roll the ball down the ramp, starting from the top, at rest, 10 times. See how many of the 10 trials landed in the ellipse!

You are *not allowed* to do a trial run for a particular ball. You may, however, do the experiment again as many times as you want as long as you use a ball with a significantly different density or radius each time. To get the maximum number of points for a particular ball, all 10 hits must land inside the ellipse, and the ellipse must be as small as possible while still containing all 10 hits. See the examples in Figure 2.

To hand in for activity 1

- Values of h and H.
- For each ball, *on its paper* showing the predicted ellipse and the 10

hits: a description of the ball, plus the calculations and reasoning for location of the center of the ellipse, and for the shape of the ellipse.

Notes on grading

- You must hand in *all* trials, though the one with the lowest score will be disregarded.
- For each trial, no credit = 10 points, full credit = 20 points.
- Your final score will be the average of all trials, after the one with the lowest score is dropped.
- You may get some extra credit, whatever your average score is, depending on the clarity of your presentations for each ball. As always, you can compensate for things not not working out perfectly by analyzing what went wrong, and telling me what you learned from your less than perfect trials.
- All this refers just to the trials; your overall grade on the lab depends on much more than all this "credit" business.

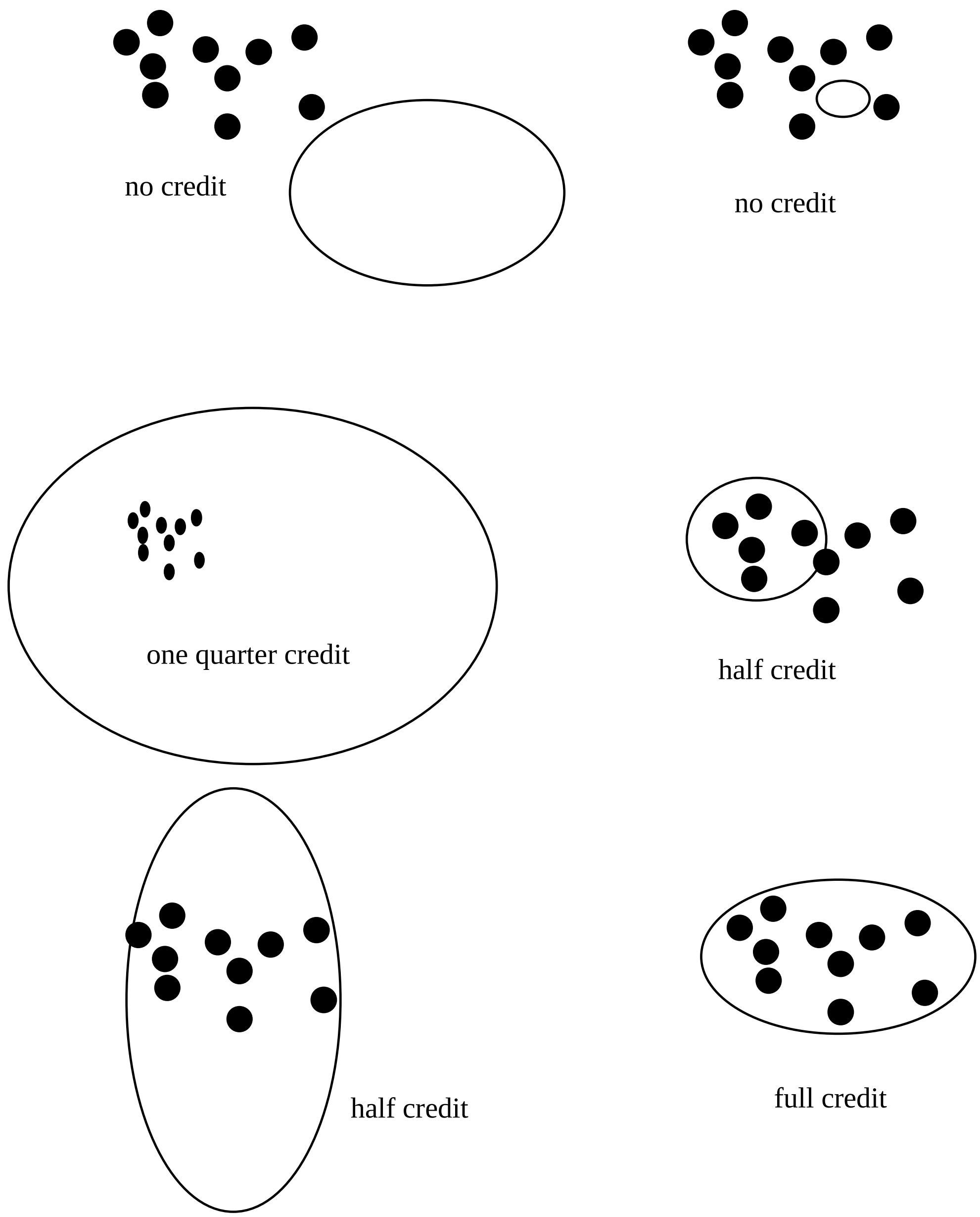

Figure 1: Examples of ellipses and trials.