

# College Physics I

## Lab 8: Cooling

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### Relationships

Many mathematical relationships arise in physics. For example, you know that for constant acceleration, the velocity  $v$  depends linearly on time  $t$ :  $v_f = v_i + at$  where  $v_i$  and  $a$  are constants. Thus, if you graphed  $v$  versus  $t$  for an object undergoing constant acceleration, you would expect a straight line. Likewise, for constant acceleration starting from rest, the position  $x$  is proportional to the square of the time:  $x_f = x_i + \frac{1}{2}at^2$ , where  $a$  is a constant. If you graphed  $x$  versus  $t$  for an object undergoing constant acceleration, you'd expect to see a curve—a parabola. If you wanted to see a straight line, you would have to graph  $x$  versus  $t^2$ .

A relationship you may not be familiar with, but which is actually more important in describing our world than the two examples above, is when one variable depends on the number  $e$  raised to the power of the other variable. If  $f$  is the dependent variable and  $y$  is the independent variable, then such a relationship would look like this:

$$f = f_0 e^{\frac{y}{y_0}},$$

where  $f_0$  and  $y_0$  are just constants inserted to make sure the units on the left match up with the units on the right.  $e$  is unitless, and whatever is in the exponent must also be unitless.

Such a relationship is called an *exponential relationship*. Exponential relationships occur whenever the rate at which something changes depends

on however much of that something there is. For example, if you want to make simple model describing the amount of bacteria there is in a petri dish at a particular time, the amount would depend on time exponentially, since the rate at which new bacteria appear is proportional to how many there already are.

You may have noticed that when you take a hot drink outside on a cold day, it cools down rapidly at first, and then more slowly as the temperature of the drink gets closer and closer to the temperature of the surroundings. This is characteristic of an exponential relationship—the rate at which the temperature of the drink changes at a particular moment depends on the current temperature difference between the drink and the surroundings at that particular moment. Let  $\Delta T$  be the temperature difference between the drink and the surroundings:  $\Delta T = T_{\text{drink}} - T_{\text{surroundings}}$ . Then a simple model for cooling might be:

$$\Delta T = \Delta T_0 e^{-\frac{t}{\tau}}$$

where  $\Delta T_0$  is the temperature difference at time zero, and  $\tau$  is a constant with units of time. Assuming our model is correct, if you graph  $\Delta T$  versus  $t$ , you will not get a straight line. There is, on the other hand, something you could graph in this situation which would give a straight line:

$$\begin{aligned}\Delta T &= \Delta T_0 e^{-\frac{t}{\tau}} \\ \frac{\Delta T}{\Delta T_0} &= e^{-\frac{t}{\tau}} \\ \ln\left(\frac{\Delta T}{\Delta T_0}\right) &= -\frac{t}{\tau} \\ \ln \Delta T - \ln \Delta T_0 &= -\frac{t}{\tau} \\ \ln \Delta T &= \ln \Delta T_0 - \frac{t}{\tau}\end{aligned}$$

This means that if you graph the natural log of the temperature difference versus time,  $\ln \Delta T$  versus  $t$ , you should get a straight line, the slope of which is equal to  $-1/\tau$ . Furthermore, this line should cross the “ $\ln \Delta T = 0$ ” axis at the value  $\ln \Delta T_0$ . Here,  $\tau$  is a time constant which has to do with the properties of the object which is cooling, and of the interface between that object and the surroundings. Let’s call  $\tau$  the time constant for the system. If  $\tau$  is very big, than it will take a long time for the object to cool. If  $\tau$  is very small, then the object will cool very quickly. But in both cases the object will cool exponentially!

## Goal

In this lab, you will test the model given above for a cooling object. You will monitor the temperature of an object at various times as it cools. First, you will see if the model above is any good for this situation by comparing what you observe with what you would expect from our model. Then, from those data, using the ideas explained above, you will determine the time constant  $\tau$  for this particular object in this particular situation.

## Activity 1: Make sure you understand

To make sure you understand the model described above, answer the following questions before you take any data. Assume that our model for cooling is correct. Suppose that at time  $t = 0$ , the temperature of the object was  $100^\circ\text{C}$ . Furthermore, suppose that the temperature of the surroundings was  $20^\circ\text{C}$ :

1. What is  $\Delta T_0$ ?
2. As time increases from  $t = 0$ , will  $\Delta T$  increase, decrease, or stay the same?
3. After a very, very long time, what will  $\Delta T$  be, for all intents and purposes?
4. Just to get a feel for what an exponential function looks like, make qualitative graphs of the functions  $f_1 = e^y$  and  $f_2 = e^{-y}$ .
5. Based on your graphs from question 4, what is  $f_1$  when  $y \rightarrow \infty$ ? What is  $f_2$  when  $y \rightarrow \infty$ ? Would  $f_1$  or  $f_2$  best describe population growth, assuming there was no war, famine, sickness, birth control, etc.?

## To hand in for activity 1

Answers to all questions.

## Activity 2: Heat the object

Take an aluminum cylinder with a hole in it, and place it in a beaker with enough water in it to cover the cylinder. Place the beaker on a hot plate, turn the hot plate up to high, and wait for the water to boil. This should take about ten minutes.

### To hand in for activity 2

*Nothing.*

## Activity 3: Monitor the temperature of the object as it cools

Check the room temperature in the region where you are going to let your object cool. Call it  $T_{\text{surroundings}}$  and write it down. When the water is boiling, turn off the hot plate and unplug it. Using tongs, carefully pour the boiling water into a sink. Then carefully “pour” the object onto a paper towel at your lab area. Use the tongs to make sure that the object has its hole facing up, and put the thermometer into the hole. The temperature,  $T_{\text{object}}$ , should rise to a maximum amount, and then start dropping. After it has started dropping, set the stopwatch at zero, and at that moment note the temperature of the thermometer. Henceforth, note time on the stopwatch every time the thermometer reads an exact degree; e.g.  $93.0^{\circ}\text{C}$ ,  $92.0^{\circ}\text{C}$ , etc. Do this for approximately twenty-five minutes.

### To hand in for activity 3

All measurements:  $T_{\text{surroundings}}$ ,  $T_{\text{object}}$  and  $t$  for each data point.

## Activity 4: Calculate derived quantities from your data

From your data, find  $\Delta T_0$ , and  $\Delta T$  for each data point.

**To hand in for activity 4**

All quantities calculated from the initial data. It would be best if the values from activities 3 and 4 were presented together in one easy to follow chart.

**Activity 5: Graphical interpretation**

Graph  $\Delta T$  versus  $t$  from the data you collected. Is the shape of the graph what you expected? Graph  $\ln \Delta T$  versus  $t$ . Assuming it is a straight line, find the slope of the best-fit line, and from that find  $\tau$  for this system.

**To hand in for activity 4**

Two graphs, and result for  $\tau$ .