College Physics I: Significant Figures

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1 Significant Figures in Simple Situations

Especially in reporting lab results, you will be expected to pay attention to significant figures (sig figs). The best way to explain is to give some examples. If something is not entirely clear, please see your instructor for a more detailed explanation.

1.1 How do you tell how many significant figures a number has?

The following numbers have 3 significant figures:

- .00356
- .00300
- 3.00×10^5
- 512
- 400.
- 0.123

These numbers are ambiguous as to the number of significant figures:

- 320 (could be two or could be three)
- 500 (could be one, two or three)

Note that the number "320." is *not* ambiguous—it has 3 significant figures. The best way to make sure the numbers you write are not ambiguous is to *use scientific notation*—if there is any doubt, write your number as "3.20×10^2."

1.2 Taking data in an experiment

When taking data in an experiment, always write your measurement to the number of digits you measure, no more and no less. For example, suppose you are measuring the length, x, of a track, with a meter stick marked to the mm place. You are able to estimate to a tenth of a mm, but certainly not to a hundredth. If you find the track to be exactly 50 cm to within the precision of your tools, then "$x = 50$ cm" is incorrect, because it does not show the full precision of the measurement. "$x = 50.000$ cm" is also incorrect, because it shows a precision to the hundredth of a mm, which is not what you measured. The correct way to write this measurement is "$x = 50.00$ cm."

1.3 Multiplying and dividing numbers

When you multiply or divide numbers, the number with the fewest significant figures decides how many significant figures the final result has. So $3.97 \times 4 = 20$ (1 sig fig, since the 4 only has 1 sig fig). It is not 16, not 15.88 and not 15.9, unless you know that the 4 is exact. *If* the 4 is exact, then the result is 15.9 (to match the 3.97), not 15.88. At the end of a calculation, always report to the correct number of sig figs. If you are doing an intermediate calculation where the result will be used to find some other result, then you should keep one extra digit to avoid rounding error, but be sure to round to the appropriate place when presenting your final result. Some more examples, assuming that none of the numbers given are exact:

- $39 \times 29/47 = 24$,
- $25.00/5.000 = 5.000$,
- $(3.0 \times 10^{-5}) \times (4.79 \times 10^{9}) = 1.4 \times 10^{5}$.

1.4 Adding and subtracting numbers

When adding and subtracting numbers, the largest decimal place in which a least sig fig lies determines the decimal place of the least sig fig of the final result. Here are some examples, assuming that none of the numbers given are exact:

- $7 - 5 = 2$,
- $7 + 5 = 12$,
- $7.5 - 5.5 = 2.0$ (*not* 2),
- $7.6 - 7.3 = .3$ (*not* .30),
- $7.0 - .1 = 6.9$,
- $7 - .1 = 7$ (*not* 6.9),
- $(4.79 \times 10^{9}) - (4.73 \times 10^{9}) = .06 \times 10^{9} = 6 \times 10^{7}$.

2 More Complicated Situations

NOTE: The material in this section is *not required* for College Physics, but College Physics students who have had Calculus may find it interesting.

Assuming f is some function of x, you want to see how a change in x translates to a change in f. If $f(x) = x + c$ (where c is a constant), then however many units you change x, you have changed f by exactly that many units. This explains example 1.4 (adding and subtracting numbers). If $f(x) = cx$, then f changes by whatever proportion x has changed; e.g.

if x changes by 1%, then f will change by 1%. This explains example 1.3 (multiplying and dividing numbers). Summarizing:

- If $f(x) = x + c$, then $\Delta f = \Delta x$.
- If $f(x) = cx$, then $\Delta f/f = \Delta x/x$.

Can we work out a general rule? Sure. Since $\Delta f/\Delta x$ is just df/dx, $\Delta f = (df/dx)\Delta x$. Let's apply this to a few common situations:

- If $f(x) = cx^n$, then $\Delta f = cnx^{n-1}\Delta x = (nf/x)\Delta x$. Therefore, $\Delta f/f = n\Delta x/x$.
- If $f(x) = \ln(x)$ then $\Delta f = (1/x)\Delta x$. Therefore, $\Delta f = \Delta x/x$ (the uncertainty in f is the percent uncertainty in x!).
- If $f(x) = e^x$, then $\Delta f = e^x\Delta x$. Therefore, $\Delta f/f = \Delta x$.
- If $f(x) = \sin(x)$, then $\Delta f = \cos(x)\Delta x$.
- If $f(x) = \cos(x)$, then $\Delta f = -\sin(x)\Delta x$.

As you can see, there is no one simple rule for handling significant figures in these more complicated situations. That is why any attempt to correctly handle sig figs in all situations can get pretty complicated—basically you have to propagate uncertainty using the general rule given above. Many texts imply that these more complicated situations can be handled by the simple rules introduced in section 1. For example, Cutnell and Johnson is full of example calculations like this: $\sin(87.6°) = .999$, implying that 3 sig figs in an angle translates to three sig figs in the sine of the angle, which is nonsense. We won't usually bother with sig figs in these more complicated situations; we are busy enough trying to learn the basic physics. The one exception is in the lab—there tracking how uncertainties in your measurements translate to uncertainties in your final result is of the utmost importance.