Solutions to Exam 1; Phys 185

1. (10 points) The equation $\sum \vec{F} = m\vec{a}$ is applicable

(a) Always

(b) Only when $|\vec{a}|$ is constant

(c) For all forces except the spring force

(d) Only if all forces cancel each other out

(e) In all cases except for non-vector forces

2. (10 points) A water strider can walk on the surface of a pond because

(a) Its weight is zero

(b) It has constant acceleration

(c) It has negative mass

(d) Surface tension holds it up

(e) The normal force cancels out its mass

3. (10 points) Planets such as Uranus and Neptune are not visible to the naked eye. How did modern astronomers get their first clues about their existence?

(a) They were accidentally sighted by new and powerful telescopes

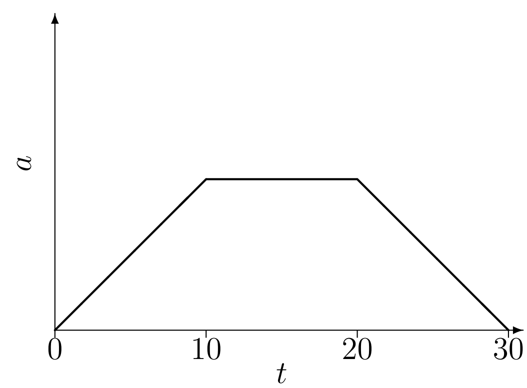
(b) Einstein's new theories of gravity held that each star has at least 8 planets

(c) Orbits of the visible planets deviated from what was calculated from $\sum \vec{F} = m\vec{a}$

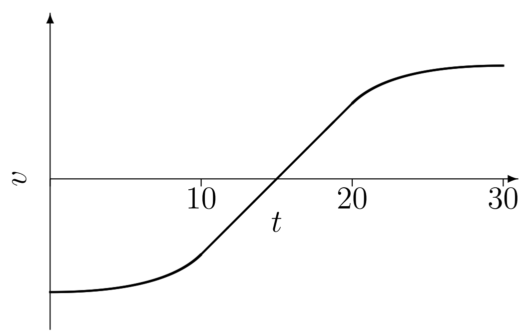
(d) Predictions for the tension force between the sun and the planets were falsified

(e) New astrological effects were needed to account for extremes of human personality

4. (10 points) The acceleration vs. time graph for a moving object is as given. Which of the following is a possible velocity vs. time graph for the same object?



(c)



5. (15 points) You have two racing cars that are right next to each other when you first look at them. They travel on a straight track with no turns. Car 1 moves at a constant velocity of 7.5 m/s. Car 2 has an initial velocity of 11 m/s, but is losing speed at a rate of 1.6 m/s^2 due to a flat tire. How far will the cars have traveled from where you first observed them when car 1 overtakes car 2?

Answer: Setting the origin of the x axis where the cars are first together, the positions of the cars are given by

$$x_1 = v_{i1}\Delta t = 7.5\Delta t$$

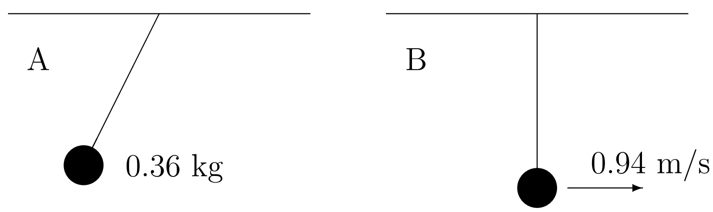
$$x_2 = v_{i2}\Delta t + \frac{1}{2}a_2(\Delta t)^2 = 11\Delta t - 0.8(\Delta t)^2$$

Overtaking means $x_1 = x_2$ once again. Therefore

$$7.5\Delta t = 11\Delta t - 0.8(\Delta t)^2$$

This is solved by $\Delta t = 0$ (the beginning) and $\Delta t = 4.4$ s, the overtaking. To find the distance, plug this into the equation either for x_1 or x_2, getting $x = 33$ m.

6. (20 points) You have a pendulum swinging back and forth: a 0.36 kg mass attached to the ceiling by a piece of string 0.55 m long.



(a) Draw a free body diagram showing the forces *on* the mass when it is swinging, and it is at the lowest height during the swing (diagram B). Draw your forces such that the length of your arrows indicates which of the forces has a larger magnitude.

Answer: A tension force $\vec{T}$ straight up, and the weight $\vec{w}$ straight down. $T > w$.

(b) What are the reaction pairs to the forces described in (a)—the forces *by* the swinging mass on other objects? What are the objects they act upon? Draw a separate diagram indicating these reaction forces.

Answer: The mass pulls with a force of magnitude T straight down, acting on the string. And the mass pulls on the earth with a force of magnitude w, straight up.

(c) Say the pendulum mass moves at a speed of 0.94 m/s when at its lowest height. Determine what the tension in the string is at this point.

Answer: Use $\sum \vec{F} = m\vec{a}$:

$$\sum F_x = 0$$

$$\sum F_y = T - mg = m\frac{v^2}{r}$$

Therefore

$$T = m\left(g + \frac{v^2}{r}\right) = 4.1 \text{ N}$$

7. (15 points) You're in a space station, in a weightless environment. Assume you have non-electronic equipment of the sort available to you in the labs you have done so far: meter sticks, stopwatch, string, known masses, springs, centripetal force apparatus, and so forth.

(a) Describe an experiment that would allow you to measure the mass of ordinary objects (not overly large or small).

Answer: There are multiple methods to do this.

One is to launch a mass from a spring compressed to a standard length, and use the stopwatch to measure how long it takes to pass the one meter mark. Then repeat the experiment with known masses, and see what known mass it takes to equal the same time. The principle here is that the spring will exert an equal force on everything, so the acceleration during the launch also depend on the mass. This means the initial velocity leaving the spring will depend on the mass.

Another method is to attach the mass to the centripetal force apparatus. Since the force by an equally extended spring is the same, and is equal to mv^2/r for the circle with radius r, the period of the motion will depend on the mass moving in a circle. You measure the period with a stopwatch, r with a meter stick. You can either use the equations in your centripetal force lab to get the mass, or use known masses to find what will give you an equal period.

(b) What would be the most important aspects of your experimental design that limit the accuracy of your mass measurements?

Answer: The timing with the stopwatch is likely to include the greatest error, due to human reaction time. Whatever the method chosen, it is best to do multiple measurement, such as measuring the period for 50 rotations, not just 1.

8. (20 points) The magnitude of the drag force is described by the equation $D = \frac{1}{2}C_D\rho Av^2$. Its direction is opposite to the direction of movement. Using this,

(a) Explain why objects dropped from a large height will not accelerate at g forever, but will instead attain a constant "terminal velocity." (*Hint:* Identify the forces on a falling object, and use $\sum \vec{F} = m\vec{a}$.) For a **bonus 5 points extra**: Find an equation for v_T, the terminal velocity of an object.

Answer: In free fall, $\vec{D}$ and the weight $\vec{w}$ are the only forces. These oppose each other, with $\vec{D}$ upward and $\vec{w}$ downward. The magnitude w remains constant. So the total force in the y direction is

$$\sum F_y = D - w = \frac{1}{2}C_D\rho Av^2 - mg$$

Note that as the object accelerates downward, its speed v will increase, and hence so will the drag force D. Eventually the object will reach a speed v_T such that the drag cancels out the weight, leaving $\sum F_y = 0$. The terminal velocity is

$$v_T = \sqrt{\frac{2mg}{C_D\rho A}}$$

Note that once an object reaches the terminal velocity, its acceleration is zero, and therefore its velocity will no longer change.

(b) Use the equation for D to explain why the terminal velocity for a person with a parachute is much less than the terminal velocity for a person falling without a parachute.

Answer: The difference between a person with a parachute and one without is C_D and A. The parachute increases the drag coefficient and the cross-sectional area, leading to a smaller v_T.

(c) Why do predatory marine animals, such as fish, have a very streamlined, hydrodynamic shape, while fast land predators, like a cheetah, are not very aerodynamic? Explain this, again referring to the equation for D.

Answer: In water, the fluid density ρ is much higher than in air, and hence the drag forces an animal has to contend with are much larger. By streamlining and presenting a small A in the direction of motion, fish reduce the drag force holding them back. For a cheetah, aerodynamics is a much less significant consideration, since air has a very low density.

9. (15 points) On the surface of the moon, objects weigh one sixth of what they weigh on earth. In other words, on the moon, $w = mg'$, where g' is the acceleration due to gravity on the moon, and this is related to the acceleration due to gravity on earth by $g' = g/6$. You can also measure the radius of the moon to be 1.74×10^6 m. Find the mass of the moon.

Answer: Set the gravitational force equal to mg':

$$mg' = G\frac{m_m m}{r_m^2}$$

Therefore,

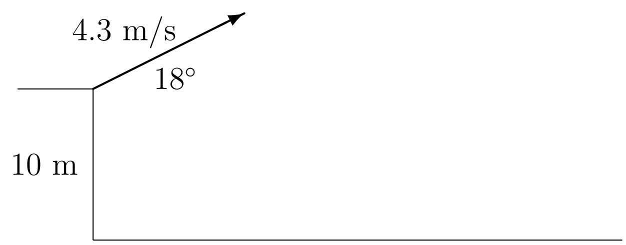
$$\frac{g}{6} = G\frac{m_m}{r_m^2}$$

With some algebra,

$$m_m = \frac{g}{6}\frac{r_m^2}{G}$$

Putting in the numbers, we get $m_m = 7.41 \times 10^{22}$ kg. Which is about right.

10. (20 points) You're on the surface of the moon, with the acceleration due to gravity of $g' = g/6$. There is no air on the moon, and therefore no air resistance. You stand at the edge of a crater that is 10.0 m deep, and kick a rock into the crater. Say you give the rock an initial speed of 4.3 m/s, and an angle of 18° with the horizontal. At what horizontal distance from the crater edge will the rock hit the bottom of the crater?



Answer: Choose the coordinate origin to be at the launching point. This means the initial values are:

$$x_i = 0 \quad y_i = 0 \quad v_{ix} = v\cos\theta = 4.1 \text{ m/s} \quad v_{iy} = v\sin\theta = 1.3 \text{ m/s}$$

As the rock hist the bottom of the crater, the final values are

$$x_f = 80 \text{ m} \qquad y_f = -10 \text{ m}$$

The accelerations are

$$a_x = 0 \qquad a_y = -g' = -1.6 \text{ m/s}^2$$

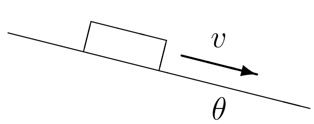
We use the equations

$$x_f = x_i + v_{ix}\Delta t \qquad y_f = y_i + v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

We can use the second equation to solve for Δt. We pick the positive root of the quadratic equations, for $\Delta t = 4.44$ s. Putting this into the equation for x_f, we get

$$x_f = 18 \text{ m}$$

11. (20 points) You're back on earth, with acceleration due to gravity g. You tilt a table at an angle θ. You then slide a book with mass m down the table, noting that it slides at a *constant* speed v. If the coefficient of kinetic friction between the table and the book is μ_k, find an equation for θ.



Answer: Constant velocity means $\vec{a} = 0$. Therefore $\sum \vec{F} = 0$. Put in the weight $\vec{w}$ with $w = mg$, the normal force $\vec{n}$, and the friction $\vec{f_k}$ with $f_k = \mu_k n$. Using titled coordinate axes, we have

$$w_x = w\sin\theta = mg\sin\theta \qquad w_y = -w\cos\theta = -mg\cos\theta$$

$$n_x = 0 \quad n_y = n \qquad f_{kx} = -f_k = -\mu_k n \quad f_{ky} = 0$$

Adding up components,

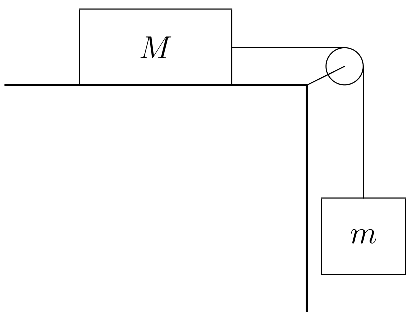
$$\sum F_x = mg\sin\theta - \mu_k n = 0$$

$$\sum F_y = -mg\cos\theta + n = 0$$

The second equation gives $n = mg\cos\theta$, which we can substitute in the first. After some cancellations, we get

$$\mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta \quad \Rightarrow \quad \theta = \tan^{-1}\mu_k$$

12. (20 points) Here is a way to measure μ_s, the coefficient of static friction between an object and a horizontal surface. You tie a length of string to the object resting on the surface, let the string go over a pulley over the edge of the surface, and add masses to a mass holder at the other end of the string.



(a) Describe how you would conduct this measurement.

Answer: Add masses gradually to the mass holder, until the total mass hanging is just enough to get the object moving.

(b) Say the mass of the object is M. If, when you have achieved the conditions you want in order to measure μ_s, the total mass you add to the end of the string is m, find an equation for μ_s.

Answer: Just before movement, when the friction force is at its maximum, the acceleration remains zero. The sum of the forces on the object is

$$\sum F_x = T - f_s = T - \mu_s n = 0$$

$$\sum F_y = n - Mg = 0$$

On the hanging masses, there are no forces with x components. Summing the rest,

$$\sum F_y = T - mg = 0$$

These give us $T = mg$ and $n = Mg$. Substituting these in the first equation, we get

$$mg - \mu_s Mg = 0$$

Solving,

$$\mu_s = \frac{m}{M}$$

13. (15 points) Give examples of situations where the following obtains. Draw a diagram for each. If no such situation is possible, explain why.

(a) The total force on an object is in the same direction as its motion

Answer: You push an object in a straight line, and it accelerates forward—its velocity increases.

(b) The direction of the total force is opposite to the direction of motion

Answer: You apply a braking force to an object moving in a straight line. It decelerates—its velocity decreases.

(c) The direction of the total force is perpendicular to the direction of motion

Answer: Uniform circular motion.

(d) The direction of the total force is perpendicular to the direction of the acceleration

Answer: Impossible. Since $\sum \vec{F} = m\vec{a}$, and m is always positive, the direction of the total force is always the same as the direction of acceleration.