
Homework Solutions # 6 (Liboff Ch. 7 & 8)

7.37 Use the 1D form, $\partial\rho/\partial t + \partial J/\partial x = 0$, and go through the derivation in equations 7.102 to 7.105. You will get

$$\frac{\partial\psi^*\psi}{\partial t} - \frac{i\hbar}{2m} \left(\psi^* \frac{\partial^2\psi}{\partial x^2} - \psi \frac{\partial^2\psi^*}{\partial x^2} \right) + \frac{i}{\hbar} \psi^*\psi(V - V^*) = 0$$

Since $V \neq V^*$, the last term does not vanish, and the continuity equation doesn't work.

The deeper reason it does not now work is not really given in Liboff, so it can be hard to figure out. It is that the density ρ and current density J must depend on ψ and ψ^* *only*, and not on other functions such as V . More formally, J must be a functional, $J[\psi, \psi^*]$ only, and if the potential term does not vanish, it ends up depending on V .

7.40 The incoming wave is $\psi_{\text{inc}} = Ae^{ik_1x}$, where the charge density is $|A|^2 = 10^{15}$ e/m, and $\hbar^2k_1^2/2m_e = 100$ eV. Use the electron mass $m_e = 0.511$ MeV/ c^2 , and the current becomes

$$J_{\text{inc}} = \frac{\hbar k_1}{m_e} |A|^2 = 5.94 \times 10^{21} \text{ e/s} = 950 \text{ A}$$

Use table 7.2 for the transmission coefficient, with $V/E = 1/2$,

$$T = \frac{4\sqrt{1 - V/E}}{[1 + \sqrt{1 - V/E}]^2} = 0.97$$

So

$$J_{\text{trans}} = 0.97J_{\text{inc}} = 5.77 \times 10^{21} \text{ e/s} = 923 \text{ A}$$

$$J_{\text{refl}} = 0.03J_{\text{inc}} = 1.78 \times 10^{20} \text{ e/s} = 27 \text{ A}$$

Sending a beam of electrons through a slit into a large capacitor would work.

7.45 Resonant maxima occur for $E > V$. Using table 7.2, T must be a maximum, which happens when $\sin(2k_2a) = 0$, or $2k_2a = n\pi$. The third maximum is at $n = 3$. So we solve for V in $\frac{2a}{\hbar} \sqrt{2m_e(E - V)} = 3\pi$, with $2a = 4.5 \times 10^{-10}$ m and $E = 100$ eV. We get $V = 83.4$ eV.

7.57 The wavefunction comes in three parts,

$$\begin{aligned}\psi_I &= Ae^{ik_1x} + Be^{-ik_1x} \\ \psi_{II} &= Ce^{\kappa x} + De^{-\kappa x} \\ \psi_{III} &= Fe^{k_3x}\end{aligned}$$

where

$$\frac{\hbar^2 k_1^2}{2m} = E \quad \frac{\hbar^2 \kappa^2}{2m} = V - E \quad \frac{\hbar^2 k_3^2}{2m} = E + \alpha V$$

The boundary conditions at $x = \pm a$ are

$$\begin{aligned}Ae^{-ik_1a} + Be^{ik_1a} &= Ce^{-\kappa a} + De^{\kappa a} \\ ik_1(Ae^{-ik_1a} - Be^{ik_1a}) &= \kappa(Ce^{-\kappa a} - De^{\kappa a}) \\ Ce^{\kappa a} + De^{-\kappa a} &= Fe^{k_3a} \\ \kappa(Ce^{\kappa a} - De^{-\kappa a}) &= ik_3Fe^{k_3a}\end{aligned}$$

We solve for T , and after a lot of algebra,

$$T = \frac{k_3}{k_1} \left| \frac{F}{A} \right|^2 = \frac{4k_1k_3\kappa^2}{\kappa^2(k_1 + k_3)^2 + (k_3^2 + \kappa^2)(k_1^2 + \kappa^2) \sinh(2\kappa a)}$$

8.7 The eigenfunctions of the semi-infinite potential well are just those for the finite one, with the extra boundary condition that $\phi(0) = 0$. This is satisfied by the $x > 0$ halves of the odd-parity solutions to the finite well, now multiplied by $\sqrt{2}$ to make the normalization come out to 1 rather than $\frac{1}{2}$. Sketched, these look like the right half of Figure 8.4b, and other right halves of sine functions with one and two zeros in $0 < x < a$. The semi-infinite well, lacking the even parity solution, will have a higher ground state energy. The eigenstates you sketched are *not*, of course, eigenstates for the finite well—you've set $\psi = 0$ for $x < 0$.

8.22 The standing-wave eigenfunctions are either odd or even. So their x -derivative will have the opposite parity, and $\langle p \rangle = 0$ because

$$-i\hbar \int_{-\infty}^{\infty} dx \psi^* \frac{\partial \psi}{\partial x} = 0$$

as the integrand is an (even \times odd) = (odd) function.

8.26 6 eV corresponds to $\nu = E/h = 1.45 \times 10^{15}$ Hz. This is the near ultraviolet.