
Homework Solutions # 6 (Liboff Ch. 7)

7.37 Use the 1D form, $\partial\rho/\partial t + \partial J/\partial x = 0$, and go through the derivation in equations 7.102 to 7.105. You will get

$$\frac{\partial\psi^*\psi}{\partial t} - \frac{i\hbar}{2m} \left(\psi^* \frac{\partial^2\psi}{\partial x^2} - \psi \frac{\partial^2\psi^*}{\partial x^2} \right) + \frac{i}{\hbar} \psi^*\psi(V - V^*) = 0$$

Since $V \neq V^*$, the last term does not vanish, and the continuity equation doesn't work.

The deeper reason it does not now work is not really given in Liboff, so it can be hard to figure out. It is that the density ρ and current density J must depend on ψ and ψ^* *only*, and not on other functions such as V . More formally, J must be a functional, $J[\psi, \psi^*]$ only, and if the potential term does not vanish, it ends up depending on V .

7.40 The incoming wave is $\psi_{\text{inc}} = Ae^{ik_1x}$, where the charge density is $|A|^2 = 10^{15}$ e/m, and $\hbar^2k_1^2/2m_e = 100$ eV. Use the electron mass $m_e = 0.511$ MeV/ c^2 , and the current becomes

$$J_{\text{inc}} = \frac{\hbar k_1}{m_e} |A|^2 = 5.94 \times 10^{21} \text{ e/s} = 950 \text{ A}$$

Use table 7.2 for the transmission coefficient, with $V/E = 1/2$,

$$T = \frac{4\sqrt{1 - V/E}}{[1 + \sqrt{1 - V/E}]^2} = 0.97$$

So

$$J_{\text{trans}} = 0.97J_{\text{inc}} = 5.77 \times 10^{21} \text{ e/s} = 923 \text{ A}$$

$$J_{\text{refl}} = 0.03J_{\text{inc}} = 1.78 \times 10^{20} \text{ e/s} = 27 \text{ A}$$

Sending a beam of electrons through a slit into a large capacitor would work.

7.45 Resonant maxima occur for $E > V$. Using table 7.2, T must be a maximum, which happens when $\sin(2k_2a) = 0$, or $2k_2a = n\pi$. The third maximum is at $n = 3$. So we solve for V in $\frac{2a}{\hbar} \sqrt{2m_e(E - V)} = 3\pi$, with $2a = 4.5 \times 10^{-10}$ m and $E = 100$ eV. We get $V = 83.4$ eV.

7.57 The wavefunction comes in three parts,

$$\psi_I = Ae^{ik_1x} + Be^{-ik_1x}$$

$$\psi_{II} = Ce^{\kappa x} + De^{-\kappa x}$$

$$\psi_{III} = Fe^{k_3x}$$

where

$$\frac{\hbar^2 k_1^2}{2m} = E \quad \frac{\hbar^2 \kappa^2}{2m} = V - E \quad \frac{\hbar^2 k_3^2}{2m} = E + \alpha V$$

The boundary conditions at $x = \pm a$ are

$$Ae^{-ik_1a} + Be^{ik_1a} = Ce^{-\kappa a} + De^{\kappa a}$$

$$ik_1(Ae^{-ik_1a} - Be^{ik_1a}) = \kappa(Ce^{-\kappa a} - De^{\kappa a})$$

$$Ce^{\kappa a} + De^{-\kappa a} = Fe^{k_3a}$$

$$\kappa(Ce^{\kappa a} - De^{-\kappa a}) = ik_3Fe^{k_3a}$$

We solve for T , and after a lot of algebra,

$$T = \frac{k_3}{k_1} \left| \frac{F}{A} \right|^2 = \frac{4k_1k_3\kappa^2}{\kappa^2(k_1 + k_3)^2 + (k_3^2 + \kappa^2)(k_1^2 + \kappa^2) \sinh(2\kappa a)}$$