Homework Solutions # 7 (Liboff Ch. 8)

8.7 The eigenfunctions of the semi-infinite potential well are just those for the finite one, with the extra boundary condition that $\phi(0) = 0$. This is satisfied by the $x > 0$ halves of the odd-parity solutions to the finite well, now multiplied by $\sqrt{2}$ to make the normalization come out to 1 rather than $\frac{1}{2}$. Sketched, these look like the right half of Figure 8.4b, and other right halves of sine functions with one and two zeros in $0 < x < a$. The semi-infinite well, lacking the even parity solution, will have a higher ground state energy. The eigenstates you sketched are *not,* of course, eigenstates for the finite well—you've set $\psi = 0$ for $x < 0$.

8.16 The $b \to \infty$ limit of equation 8.55a is, with $\cosh \kappa b \to \infty$,

$$\cos k_1 a - \frac{k_1^2 - \kappa^2}{2k_1\kappa} \sin k_1 a = 0$$

For $E \ll V$, $k_1 \ll \kappa$ (from equations 8.54 and 8.55b), so we're left with

$$2\frac{k_1}{\kappa} \cos k_1 a = \frac{k_1^2 - \kappa^2}{\kappa^2} \sin k_1 a \quad \Rightarrow \quad 0 \approx - \sin k_1 a$$

We have $\sin k_1 a = 0$, hence $k_1 a = n\pi$. The curves in Figure 8.13b all become flat lines.

8.22 The standing-wave eigenfunctions are either odd or even. So their x-derivative will have the opposite parity, and $\langle p \rangle = 0$ because

$$-i\hbar \int_{-\infty}^{\infty} dx\, \psi^* \frac{\partial \psi}{\partial x} = 0$$

as the integrand is an (even $\times$ odd) = (odd) function.

8.26 6 eV corresponds to $\nu = E/h = 1.45 \times 10^{15}$ Hz. This is the near ultraviolet.