HOW TO RESEARCH MATHEMATICS

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The purpose of this introductory guide is to help you begin to research mathematics.

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1 What is mathematical research?

Research is an act of discovery. It is also an act of creation.

The word research comes from the Middle French *recerce* or the French *recerche*, and it means, "A careful search, a close searching; a studious inquiry, a critical and exhaustive investigation or experimentation having for its aim the revision of accepted conclusions in the light of newly discovered facts."

The common assumption in the academic world is that research is a careful, studious inquiry leading to the creation of new knowledge or a critical and exhaustive exposition of already known material. There is very little misunderstanding and debate about the first case: the creation of new knowledge will always be recognized as the mark of a successful research project. As for the second case, a careful expository endeavor can qualify as research if the point of view or the organization of the material is new, or if new conclusions are drawn.

Researching mathematics is not very different from studying mathematics. The good skills you develop in studying mathematics can be put to good use in researching new

mathematics. This guide will give your some pointers on how to get started and how to carry out your research successfully.

A resource for mathematical problem solving (which is a lot of what research is about) is [1].

2 Finding a research topic

You should choose a research topic based on something you are interested in. Did you take a course that peaked your curiosity? Do you have a professor who works in an area that you'd like to find out more about? Do you have friends that are studying a topic that sounds appealing? These situations call all indicated potential areas of research for you.

3 General advice

The best advice I got about mathematical research was from my professor Donald Lewis. He said that you should always keep several irons in the fire. This means that you should work on several topics at one time, because you never know when and where you will make a breakthrough.

Other good advice comes from Fan Chung, who says, "In mathematics whatever you learn is yours and you build it up—one step at a time." I would add that in mathematics speed doesn't matter.

Another interesting piece of advice that I heard, that I have to agree is true, is to "think with a pen." This means, literally, to think with a pen in your hand and write formulas, doodle, and get feed-back from your own written words and symbols.

As you progress with your research project, keep the following pointers in mind:

- Do examples so you can understand what is going on.
- Keep asking yourself questions. Why does a particular piece of mathematics work the way it does?
- Write what you know, and update your writing as you learn more.
- Explain what you know to others (instructors and students) and listen to their feed-back. You understand best when you explain to others. Also, your audience may point out that certain things are trivial or not understandable, and they might suggest alternate proofs or other aspects of the problem that deserve to be looked at.

4 Taking basic steps

Successful mathematicians use a variety of strategies to discover mathematics, but there are some common techniques that bear fruit time and time again. These techniques are central to the mathematical process.

Data Generate examples.

Observations Look at your data and find general patterns or trends.

Conjectures Make conjectures based on your observations. What might be true in general?

Proof Now try to prove your conjectures. You may want to start with proving "small" conjectures, that is, ones concerned with details, before launching into a proof of a "grand conjecture."

Generalization Once you have obtained a proof of a significant conjecture—and now you can call your result a theorem—try to generalize your ideas. When will the same statement hold, and can you prove it?

Connections It is always useful and enlightening to see your piece of mathematics as part of the tapestry of math. What role do your finding play? How do they relate to other areas and results of mathematics?

5 Avoiding common mistakes

The most common mistakes are simply not following the good advice. Here is a list of what could go wrong and how to remedy it:

- You can't get any "good results." Go back to the basics of generating examples and looking for patterns. Your results don't have to be earth-shattering. Notice details and then try to prove that the patterns your observe are really true. Little by little, you can build up interesting results.

- You write up your work in a way that is too abstract, and your instructors and peers can't understand what you have found. You need a mathematical "reality check." Remember that in mathematics, the goal is not to make things difficult, but rather to explain and prove interesting and important mathematical phenomena. If you work is too complicated-looking, you may not have arrived at the simple mathematical truth you are aiming for. Try to simplify the statements of your results, as well as the notation and proofs. Ask yourself, what is this work about? What are the basic examples that mathematicians would be interested in knowing about? Proceed from those basic examples and keep polishing your work so it is at just the right level of complexity to get the job done.

- You discover things that are already known. This is inevitable with all mathematicians. We are excited to find a "new theorem," only to discover that another mathematician has been there before. Don't despair: you were able to discover something independently, and that is good. Keep using your talents and eventually you can discover something new.

- You are having trouble writing up your work. Again, go back to examples. Start your written work with a significant example, not trivial but not too complicated. This will help set the stage for why your work is important and interesting.

6 Using resources

What are the most important resources available to use in researching mathematics?

Mathematics exploration software It is difficult to overestimate the importance of generating data and looking for patterns in exploring mathematics. The following software can help when the computations are complicated.

- *Mathematica*
- *Maple*

Mathematics typesetting software The typesetting software of choice today is LaTeX. There are several good ways access this software.

- The company PCTeX (`www.pctex.com`) has a particularly easy-to-easy implementation of LaTeX.

Web resources The Web is an almost unlimited sea of information. The sites below may be particularly helpful.

- MathWorld (`www.mathworld.com`)
- Wikipedia (`www.wikipedia.org`)
- MathSciNet (`www.ams.org/mathscinet/`)
- JStor (`www.jstor.org`)

7 Exercising good mathematical taste

One of the most difficult to explain, but nonetheless one of the most important, aspects of mathematical research is the question of mathematical judgment, or (for want of a better word) taste. You choose your area of research because it is something that interests you, or your advisor, or both. Try to make sure that your work is something that others would be interested in too. Don't be too abstract in your theorizing, or prove results that are "all-comprehensive." Try to work with specific examples leading to well-defined results. Always subject your work to the questions, "Can I explain this to others?" and "Would others be interested?"

The issue of mathematical judgment pervades the research process and should be kept in mind at every step. An ever-present opportunity to exercise good judgment is in choosing natural notation and natural ways to write up your work. This is a good reason for writing your work as you go along—it helps you keep your effort real.

References

[1] G. Pólya. *How to Solve It: A New Aspect of Mathematical Method*. Doubleday, New York, 1957.