

**Key to Supplemental Questions
for
Science and Measurement Lectures**

1. Several students independently determine of the volume of a wood block and report its volume as $37.655(1) \text{ cm}^3$.

a. What is the average value of the block's volume? **37.655 cm^3**

b. What is the standard deviation of the block's volume? **0.001**

c. Is this a precise measurement? Explain.

The standard deviation of the volume measurement is very small, so this is a fairly precise measurement.

d. Is this an accurate measurement? Explain.

The accuracy of the measurement can not be determined, because the true or accepted value is not known.

2. The accepted atomic weight of bromine is 79.904 amu. If we determine the atomic weight of bromine to be 79.91 amu, how accurate is our result? Is it precise? Explain.

Since the true value is known, we can assess the accuracy using percent error.

$$\% \text{ error} = \left(\frac{\text{measured value} - \text{true value}}{\text{true value}} \right) \times 100$$

$$\% \text{ error} = \left(\frac{79.91 \text{ amu} - 79.904 \text{ amu}}{79.904 \text{ amu}} \right) \times 100 = \left(\frac{0.006 \text{ amu}}{79.904 \text{ amu}} \right) \times 100 = 8. \times 10^{-3}\%$$

The calculated value has a % error of only $8. \times 10^{-3}\%$. We can conclude that the result is quite accurate.

However, it is not as precise as the accepted value because the accepted value is known to at least ± 0.001 amu, while our value is only known to ± 0.01 amu.

3. Perform the following calculations and report the result to the proper number of significant figures. Write the answers in scientific notation as necessary.

$$(45.9876 + 0.0023) \times (3.54 \times 10^3) = \mathbf{1.63 \times 10^5}$$

$$\frac{8.97002 \times 10^2}{14.777} = \mathbf{60.702} \text{ (60.703 also accepted)}$$

$$0.2310 \left(\frac{2}{5} \right) \left(\frac{207.2}{1} \right) = \mathbf{19.14} \text{ (19.15 also accepted)}$$

$$5.010 \times 10^{-4} + 3.00 \times 10^{-3} = \mathbf{3.50 \times 10^{-3}}$$

4. Stibnite is a mineral from which the element antimony may be obtained. A sample of stibnite is reported to be $10.6 \pm 0.1\%$ by weight antimony at the 95% confidence limit. What is the average percent by weight of antimony in the stibnite?

The average percent by weight of antimony in this stibnite sample is 10.6%.

b. You have 250.00 g of this stibnite sample, on average how many grams of antimony do you have?

$$250.00 \text{ g stibnite} \left(\frac{10.6 \text{ g Sb}}{100.0 \text{ g stibnite}} \right) = 26.5 \text{ g Sb}$$

There are, on average, 26.5 g of antimony in 250.00 g of this stibnite sample.

c. What is the maximum amount of antimony (in grams) that you would expect to find in 250.00 g of this stibnite sample?

The maximum amount of antimony we expect to find is the upper limit of the confidence interval given in part a ($10.6 + 0.1$ or 10.7%).

$$250.00 \text{ g stibnite} \left(\frac{10.7 \text{ g Sb}}{100.0 \text{ g stibnite}} \right) = 26.8 \text{ g Sb}$$

The maximum amount of Sb we expect to find is 26.8 g.

d. If the true value for the percent by weight of antimony in the stibnite sample is 9.4%, what is the percent error for the value quoted in part a?

The definition of % error is

$$\% \text{ error} = \left(\frac{\text{measured value} - \text{true value}}{\text{true value}} \right) \times 100$$

Substituting in gives

$$\% \text{ error} = \left(\frac{10.6\% - 9.4\%}{9.4\%} \right) \times 100 = +13.\%$$

The percent error for the weight percent of Sb in the sample of stibnite is +13.%.

5. Perform the following calculations. Clearly show the correct number of significant figures in your answers.

$$\begin{array}{r} 37.695 \\ + 0.13 \\ \hline 37.83 \end{array}$$

$$\begin{array}{r} 45.9 \\ \times 9.021 \\ \hline 414. \end{array}$$

$$\begin{array}{r} 19.003 \\ - 19.000 \\ \hline 0.003 \end{array}$$

$$\begin{array}{r} 12.011 \\ \div 6.022 \times 10^{23} \\ \hline 1.995 \times 10^{-23} \end{array}$$

6. Indicate the number of significant figures in the following numbers.

0.01030 **4**

3.145×10^{-2} **4**

10. **2**

1.0020×10^4 **5**

7a. At 25.0 °C the density of liquid water is 0.997 g/cm³, but at -10.0 °C the density of solid water (ice) is 0.917 g/cm³. If a 250.0 mL sample of liquid water originally at 25.0 °C is frozen and cooled to -10.0 °C, what volume will the solid occupy?

Find the mass of water at 25 °C.

$$250.0 \text{ mL} \left(\frac{1 \text{ cm}^3}{1 \text{ mL}} \right) \left(\frac{0.997 \text{ g}}{1 \text{ cm}^3} \right) = 249.25 \text{ g}$$

This mass of water must also be present at -10.0 °C, so use it to calculate new volume.

$$249.25 \text{ g} \left(\frac{1 \text{ cm}^3}{0.917 \text{ g}} \right) \left(\frac{1 \text{ mL}}{1 \text{ cm}^3} \right) = 271.8 \text{ mL}$$

At -10.0 °C the solid water will occupy a volume of 272. mL.

b. What practical implication does your answer to part a have to freezing water in closed containers?

Water expands when it freezes. If one freezes water in a closed container, it is possible that the container will burst.

8. In SI units the prefix *yocto-* (abbreviation *y-*) denotes the multiplier 10⁻²⁴. Given this information, is it possible to have a ymole of atoms? Explain.

Do the conversion from ymole to atoms.

$$1 \text{ ymole} \left(\frac{1 \times 10^{-24} \text{ mole}}{1 \text{ ymole}} \right) \left(\frac{6.022 \times 10^{23} \text{ atoms}}{1 \text{ mole}} \right) = 0.6022 \text{ atom}$$

You can't have a ymole of atoms because this would be less than a whole atom, which according to atomic theory is impossible since an atom is indivisible.

9a. An experiment was done to determine the mass of a proton. The results for several measurements were averaged to give a mass of 1.67876 yg (for the definition of the metric prefix y-, see problem 2). If the standard deviation is calculated to be 0.00202, are all of the significant figures on the average justified? Explain.

All of the significant figures on the average are not justified because the standard deviation indicates that the uncertainty is in the 1/1000's place, not in the 1/100000's place as indicated in the average.

The number of significant figures in the average and standard deviation are not relevant.

b. How should these results (the average mass and standard deviation) be reported?

The results should be reported as 1.679(2) yg.

c. Another set of experiments gave the mass of the proton as 1.67251 yg with a standard deviation of 0.00021. From the information given this far in the problem can you determine whether this result is more precise than the first measurement? Is it more accurate? Explain.

The second set of measurements is more precise, because the standard deviation for these measurements is smaller than the standard deviation for the first set of measurements.

We can not judge the accuracy of these results, because at this point we do not know the true mass of the proton.

d. If the mass of the proton is known to be 1.67262 yg, which of the two measurements discussed in this problem is more accurate? Calculate the percent error for each measurement to support your argument.

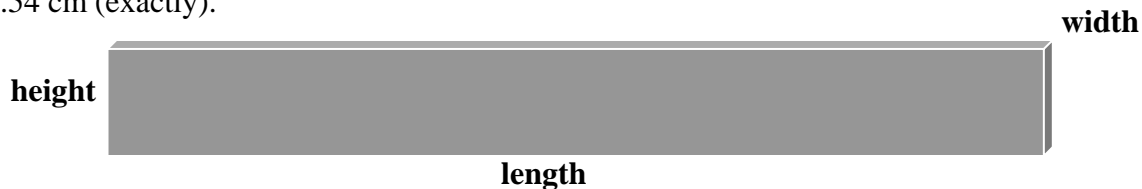
Calculate the percent error for measurement #1 and measurement #2 using the definition of percent error.

$$\%Error\#1 = \frac{1.679 - 1.67262}{1.67262} \times 100 = 0.4\%$$

$$\%Error\#2 = \frac{1.67251 - 1.67262}{1.67262} \times 100 = 7. \times 10^{-3}\%$$

The % error for the second measurement is smaller than the % error for the first measurement, so the second measurement is more accurate.

10. Aluminum foil for home use comes in packages weighing approximately 12. oz and containing 75. ft² of aluminum. If the density of aluminum is 2.70 g/cm³, what is the approximate thickness of the foil in millimeters? You are given that 1 oz = 28.4 g and that 1 in = 2.54 cm (exactly).



We can find the volume of the foil from its dimensions and from its mass and density. From this we can find the thickness, if we know the other two dimensions. The length and width of the foil are not given, but the area is. Since the area is the product of the two unknown dimensions, and this is all we really need, we can use the area in the calculation.

Calculate volume from the dimensions.

$$V = l \times w \times h = \text{area} \times h = 75. \text{ ft}^2 \times h$$

Convert area to metric units.

$$75. \text{ ft}^2 \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^2 \left(\frac{2.54 \text{ cm}}{1 \text{ in}} \right)^2 = 6.9_{677} \times 10^4 \text{ cm}^2$$

So the volume of the foil is

$$V = (6.9_{677} \times 10^4 \text{ cm}^2) \times h$$

Find the volume from the given mass and the density, but first convert mass in ounces to grams.

$$12. \text{ oz} \left(\frac{28.4 \text{ g}}{1 \text{ oz}} \right) = 3.4_{08} \times 10^2 \text{ g}$$

$$3.4_{08} \times 10^2 \text{ g} \left(\frac{1 \text{ cm}^3}{2.70 \text{ g}} \right) = 1.2_{622} \times 10^2 \text{ cm}^3$$

Since these volumes must be equal.

$$V = (6.9_{677} \times 10^4 \text{ cm}^2) \times h = 1.2_{622} \times 10^2 \text{ cm}^3$$

$$h = 1.8_{115} \times 10^{-3} \text{ cm}$$

Convert thickness in cm to mm.

$$1.8_{115} \times 10^{-3} \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) \left(\frac{1000 \text{ mm}}{1 \text{ m}} \right) = 1.8 \times 10^{-2} \text{ mm}$$

The thickness of the foil is approximately 1.8 x 10⁻² mm.

